

Practice Worksheet on Geometric Progressions and Inequalities

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Subject: Mathematics

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Long Answer Questions

1. Solve the inequality and write the solution using interval notation: $x \in (-\infty; -1) \cup (-1; 1) \cup [2; +\infty)$
2. Solve the equation for q , given that it represents the common ratio of a geometric progression with first term $a > 0$ and satisfies the equation $aq^3 + aq^2 = 2aq$. Explain the steps.
3. Explain why the line AB is tangent to the circumcircle of triangle VSK . Include the geometrical reasoning and any relevant theorems or properties.
4. Explain why it is impossible for a table to have even sums in each row and column and adhere to the given constraints. Show a proof by contradiction.
5. Derive the function $f(x)$ given that $f(0) = -1$ and it satisfies the equation $f(x) = y f(x) + 2020 f(y) + 2019$. Explain the steps taken in deriving and verifying the solution.

6. Find the solution to the quadratic equation $(q-1)(q^2+q-1) = 0$. What does this equation represent in the context of geometric progressions?

7. Explain the role of parallel lines in proving that line AB is tangent to the circumcircle of triangle VSK. Describe the geometrical relationships involved.

8. Explain how the given conditions on the row and column sums of the table lead to a contradiction, making the scenario impossible. Explain the logical steps leading to the conclusion.

Multiple Choice Questions

1. What is the solution set for the inequality: $x \in (-\infty; -1) \cup (-1; 1) \cup [2; +\infty)$

- a) $(-\infty, -1) \cup (-1, 1) \cup [2, \infty)$
- b) $(-\infty, -1] \cup (-1, 1) \cup (2, \infty)$
- c) $(-\infty, -1) \cup [-1, 1] \cup [2, \infty)$
- d) $(-\infty, -1] \cup [-1, 1] \cup (2, \infty)$

2. What type of sequence is described by the equation $aq^3 + aq^2 = 2aq$?

- a) Geometric progression
- b) Arithmetic progression
- c) Harmonic progression
- d) Fibonacci sequence

3. What condition must be met for line AB to be tangent to the circumcircle of triangle VSK?

- a) $\angle QBC = \angle BKC$
- b) $\angle QBC + \angle BKC = 180^\circ$
- c) $\angle QBC = 2\angle BKC$

d) $\sum QBC = \sum BKC/2$

4. What type of proof is used to show that the table with the given conditions is impossible?

- a) Contradiction
- b) Proof by induction
- c) Direct proof
- d) Proof by exhaustion

5. What is the derived function $f(x)$ satisfying the equation and condition $f(0) = -1$?

- a) $f(x) = 1/2020 * x - 1$
- b) $f(x) = 2020x - 1$
- c) $f(x) = -2020x + 1$
- d) $f(x) = -x + 2019$

6. What quadratic equation needs to be solved to find the common ratio of the geometric progression?

- a) $(q - 1)(q^2 + q - 1) = 0$
- b) $(q + 1)(q^2 - q + 1) = 0$
- c) $(q - 1)(q^2 - q - 1) = 0$
- d) $(q + 1)(q^2 + q - 1) = 0$

7. Which lines are crucial in proving the tangency of line AB to the circumcircle of triangle VSK?

- a) Parallel lines
- b) Perpendicular lines
- c) Intersecting lines
- d) Concurrent lines

8. What is the core reason for the impossibility of the table described?

- a) The row and column sum constraints result in conflicting inequalities

- b) The table has too many numbers
- c) The numbers in the table are not integers
- d) There is no solution to the system of equations

Answer Key

Long Answer Questions - Expected Responses

1. Solve the inequality and write the solution using interval notation: $x \notin (-\infty; -1) \cup (-1; 1) \cup [2; +\infty)$

Expected Answer: The solution set is all real numbers except for -1 and 1 , and the interval $[2, +\infty)$.

2. Solve the equation for q , given that it represents the common ratio of a geometric progression with first term $a > 0$ and satisfies the equation $aq^3 + aq^2 = 2aq$. Explain the steps.

Expected Answer: The solution involves solving a quadratic equation derived from the geometric series formula.

3. Explain why the line AB is tangent to the circumcircle of triangle VSK . Include the geometrical reasoning and any relevant theorems or properties.

Expected Answer: The line AB is tangent to the circumcircle of triangle VSK if and only if the angles $\angle QBC$ and $\angle BKC$ are equal.

4. Explain why it is impossible for a table to have even sums in each row and column and adhere to the given constraints. Show a proof by contradiction.

Expected Answer: A contradiction arises because the sum of all the numbers is both less than or equal to a value and greater than or equal to another value, which are not equal.

5. Derive the function $f(x)$ given that $f(0) = -1$ and it satisfies the equation $f(x) = y f(x) + 2020 f(y) + 2019$. Explain the steps taken in deriving and verifying the solution.

Expected Answer: The function is derived by using the given equation and the condition that $f(0) = -1$. Then, the answer is verified.

6. Find the solution to the quadratic equation $(q-1)(q^2+q-1) = 0$. What does this equation represent in the context of geometric progressions?

Expected Answer: It is a solution for quadratic equation derived from a geometric progression.

7. Explain the role of parallel lines in proving that line AB is tangent to the circumcircle of triangle VSK. Describe the geometrical relationships involved.

Expected Answer: Parallel lines create equal angles when intersected by a transversal line. This property helps prove the tangency of the line AB to the circle.

8. Explain how the given conditions on the row and column sums of the table lead to a contradiction, making the scenario impossible. Explain the logical steps leading to the conclusion.

Expected Answer: The constraints on the row and column sums create a contradiction. The argument demonstrates that such a table is mathematically impossible.

Multiple Choice Questions – Correct Answers

1. What is the solution set for the inequality: $x \in (-\infty; -1) \cup (-1; 1) \cup [2; +\infty)$

Correct Answer: $(-\infty, -1) \cup (-1, 1) \cup [2, \infty)$

2. What type of sequence is described by the equation $aq^3 + aq^2 = 2aq$?

Correct Answer: Geometric progression

3. What condition must be met for line AB to be tangent to the circumcircle of triangle VSK?

Correct Answer: $\angle QBC = \angle BKC$

4. What type of proof is used to show that the table with the given conditions is impossible?

Correct Answer: Contradiction

5. What is the derived function $f(x)$ satisfying the equation and condition $f(0) = -1$?

Correct Answer: $f(x) = 1/2020 * x - 1$

6. What quadratic equation needs to be solved to find the common ratio of the geometric progression?

Correct Answer: $(q - 1)(q^2 + q - 1) = 0$

7. Which lines are crucial in proving the tangency of line AB to the circumcircle of triangle VSK?

Correct Answer: Parallel lines

8. What is the core reason for the impossibility of the table described?

Correct Answer: The row and column sum constraints result in conflicting inequalities